J. Bus. Financ. 01 (2013) 11-16

Available Online at eSci Journals
Journal of Business and Finance
ISSN: 2305-1825 (Online), 2308-7714 (Print)
http://www.escijournals.net/JBF

ASSET-LIABILITY MANAGEMENT MODELS IN DECISION MAKING

[a]Narela Bajram, [b]Mehmet Can
[a] *Faculty of Business and Administration, International University of Sarajevo, Hrasnicka Cesta, Ilidza, Sarajevo BiH.*
[b] *Faculty of Engineering and Natural Sciences, International University of Sarajevo,Hrasnicka Cesta, Ilidza, Sarajevo BiH.*

A B S T R A C T

This paper uses an asset-liability management model to solve multi-period investment problems. The model aims to maximize the overall revenue and deal with uncertainties as well as with risks. The assumption of a linear utility function may lead to allocation of the wealth to one asset. This paper sheds some light on this issue by showing that the linear function can be a risky choice. For this purpose to solve multi-period investment problem we used two ways: first, using a piecewise linear function; and second using a non-linear utility function. The results show that the non-linear function outperform the piecewise linear function and generates better asset allocation. The problem is formulated by using the Wolfram Mathematical Programming System.

Keywords: Asset-liability management model (ALM), linear and nonlinear utility function, portfolio optimization and multi period asset allocation.

INTRODUCTION

The Asset-Liability Management (ALM) problem has crucial importance to pension funds, insurance companies and banks where business involves large amount of liquidity. Indeed, the financial institutions apply ALM to guarantee their liabilities while pursuing profit. The liabilities may take different forms: pensions paid to the members of the scheme in a pension fund, savers' deposits paid back in a bank, or benefits paid to insurers in the insurance company. A common feature of these problems is the uncertainty of liabilities and the resulting risk of underfunding. The other major uncertainty originates from asset returns. Together they constitute a nontrivial difficulty in how to manage risk in the model applied by the financial institution. The need for multi-period planning additionally complicates the problem.

Stochastic programming provides a general purpose-modeling framework, which captures the real-world features such as turnover constraints, transaction costs, risk aversion, limits on groups of assets and other consideration. However, the optimization model turns out to be intractable for the enormous number of decision variables, especially for the multi-stage problems. One of the first industrially applied models of this type was the stochastic linear program with simple recourse developed by Kusy and Ziemba in 1986. This model captured certain characteristics of ALM problems: it maximized revenues for the bank in the objective under legal, policy, liquidity, cash flow and budget constraints to make sure that deposit liability is met as much as possible. Under computational limits at the time when it was developed, this model took the advantage of stochastic linear programming so as to be practical even for the large problems faced in banks.

In this paper we demonstrate how ALM model can be applied for asset allocation in financial markets. We assume a very simple model with tree "assets"; stock A and Stock B and bonds.

Moreover, it is important for decision makers to rebalance the portfolio during the investment period as they may wish to adjust the asset allocations according to updated information on the market. The strategy which is currently optimal may not be optimal any more as the situation changes. Thus, it is important to reconsider the strategy and make the necessary changes in order to remain in the optimal position. Taking this into account, the investor is allowed to rebalance annually using the new information at the

* Corresponding Author:
Email: narela7@yahoo.com
© 2012 eSci Journals Publishing. All rights reserved.

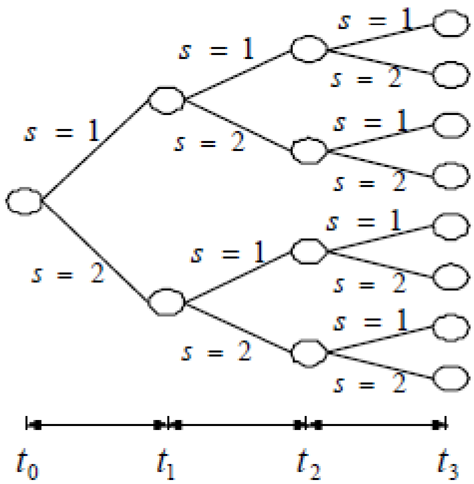

Figure 1: A scenario tree for a multi-stage stochastic program.

end of each period. In order to allow different decisions thought the investing process a multi-stage ALM model is used.

To make it easier to model, we consider the problem stage by stage and with portfolio rebalancing done at the beginning of each stage. Also, the uncertainties of asset returns are implemented with discrete distributions, in which case an event tree is used to capture the uncertainties in multiple stages throughout the whole decision process, e.g. as shown in Figure 1. The nodes at each stage represent possible future events. Asset returns, liabilities and cash deposits are subject to uncertain future evolution. Meanwhile, the asset rebalancing is done after knowing which values the asset returns and liabilities take at each node.

The paper is organized as follows: in Section 2 we provide a classification of the more recent life-cycle asset allocation models based on the type of available solutions. Section 3 describes the stochastic programming model, in particular the formulation of the objective, the optimization approach using piecewise linear utility function and nonlinear utility function, and the generation of scenarios. In Section 4, numerical results from the ALM model are compared and Section 5 concludes our study.

OVERVIEW OF ASSET ALLOCATION MODELS

The classical treatments of strategic asset allocation can be traced back to Samuelson (1969) and Merton (1969, 1971). In the light of Markowitz'(1952) paper on single-period portfolio selection, the early literature focused on conditions leading to the optimality of myopic policies, i.e., conditions under which portfolio decisions for multi-period problems coincide with those for single period problems. In addition, the lack of computing power leads to formulate models driven by the quest for closed form solutions. To achieve these objectives, rather restrictive assumptions were made, and many of these models' results turned out to be inconsistent with conventional wisdom as expressed by the so-called Samuelson puzzle: the optimal allocation does not depend on the investor's horizon and the investor with power utility who rebalances his portfolio optimally should choose the same asset allocation. This contradicts the advice obtained from many professionals in practice that investors should hold a share of risky assets because they look relatively less risky as they approach retirement (often called the age effect).

Since then, many researchers have tried to resolve this puzzle which is mainly rooted in some of the (simplifying) assumptions used in early models (fixed planning horizon, time-constant investment opportunities, no intermediate consumption, etc.).

Research in the area of life-cycle asset allocation models regained momentum in the early 1990s for two main reasons: first, a number of economic factors increased the number of people with sizeable wealth to invest (the "generation of heirs"), coupled with increased uncertainty about the security of public pension systems. Second, the enormous increase in computer power enabled the solution of models with more realistic assumptions. A number of additional features have been added to the classical models, in many cases with the goal of resolving the Samuelson puzzle: stochastic labor income, time-varying investment opportunities, parameter uncertainty (with and without learning), special treatment of certain asset classes (real estate), and habit formation, to name just the most important developments.

In contrast to other approaches in the literature using non-linear optimization (see, e.g., Blomvall and Lindberg 2002; Gondzio and Grothey 2007), we use multi-period stochastic linear programming (SLP) to solve the problem of optimal life-cycle asset allocation and consumption. This method has been explicitly chosen with the practical application of the approach in mind. Many features which are considered important for investment decisions in practice can be easily incorporated when using SLP. For example, personal characteristics of the investor can be taken into account (e.g., mortality risk, risk attitude, retirement, future cash flows for major purchases or associated with other life events). Combined with the availability of efficient solvers, this explains why the SLP approach has been successfully applied to a wide range of problems (see, e.g., Wallace and Ziemba 2005). To nest classical analytical results from this area within our model, we maximize expected utility of consumption over the investor's lifetime and expected utility of bequest rather than other objectives which can be implemented more easily (e.g., piecewise linear or quadratic penalty functions, or minimizing CVaR).

An important reference for the present paper is Campbell et al. (2003). They model asset returns and state variables as a first-order vector autoregression VAR(1) and consider Epstein–Zin utility with an infinite planning horizon. Additional assumptions include the absence of borrowing and short-sale constraints. Linearizing the portfolio return, the budget constraint, and the Euler equation, they arrive at a system of linear-quadratic equations for portfolio weights and consumption as functions of state variables. This system of equations can be solved analytically, yielding solutions which are exact only for a special case (very short time intervals and elasticity of intertemporal substitution equal to one), and accurate approximations in its neighbourhood.

The SLP used in the present paper has been applied successfully to a number of related problems. To cite only a few examples, there are applications in insurance (Cariño and Ziemba 1994, 1998; Cariño et al. 1998), and the pension fund industry (e.g., Gondzio and Kouwenberg 2001). Zenios (1999) surveys large-scale applications of SLP to fixed income portfolio management. General aspects of applying such models in a strategic asset allocation context are discussed in Ziemba and Mulvey (1998), Pflug and Swietanowski (2000), Gondzio and Kouwenberg (2001), Wallace and Ziemba (2005), and Geyer and Ziemba (2007). Particular aspects that are relevant in a life-cycle portfolio context are discussed in Geyer et al. (2007).

A MULTISTAGE MODEL: ASSET- LIABILITY MANAGEMENT

The best way to introduce multistage stochastic model is a simple asset liability management (ALM) model (Birge and Louveaux 1967). We have an initial wealth W^0 that should be properly invested in a way to meet a liability L at the end of the planning horizon H.

If possible, we would like to own a terminal wealth W^H larger than L; however, we should account properly for risk aversion, since there could be some chance to end up with a terminal wealth that is not sufficient to pay for the liability, in which case we will have to borrow some money.

A nonlinear, strictly concave utility function of the difference between the terminal wealth W_H, which is a random variable, and the liability L would do the job, and this would lead to a nonlinear programming model. In this paper we will present two alternatives of modelling the portfolio decisions for multi-period problems. As a first alternative, we can build a piecewise linear utility function like the one illustrated in Figure 2.

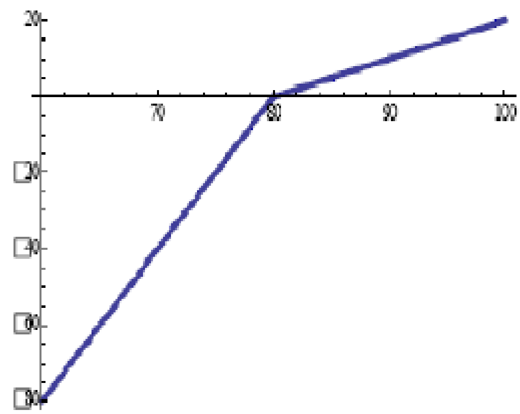

Figure 2: Piecewise Linear Utility Function.

And as a second alternative, we can build a nonlinear utility function like the one illustrated in Figure 3.

The utility is zero when the terminal wealth WH matches the liability L exactly. If the slope r penalizing the shortfall is larger than reward rate (q), this function is concave (but not strictly). The portfolio consists of a set of 3 assets. For simplicity, we assume that we may rebalance it only at a discrete set of time instants t = 1, ... , H-1, with no ransaction cost; the initial portfolio is chosen at time t = 0, and the liability must be paid at Time H.

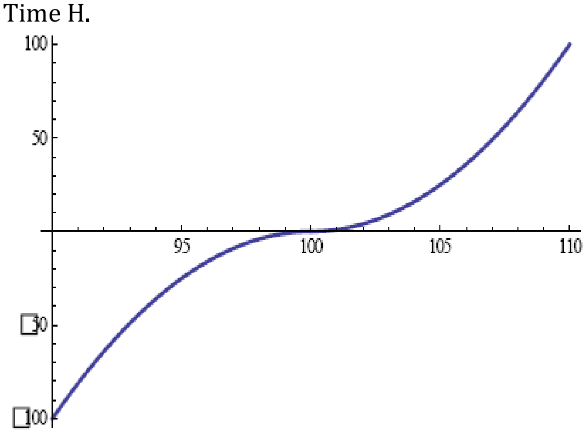

Figure.3 The Nonlinear Utility Function.

Time period t is the period between time instants t - 1 and t. In order to represent uncertainty, we may build a tree like that in Fig. 4. , which is a generalization of the two-stage tree. Each node nk in the tree corresponds to an event, where we should make some decision. We have an initial node n0 corresponding to time t = 0.

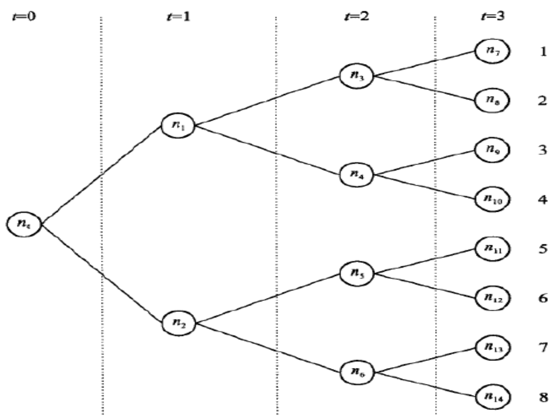

Figure 4. Scenario tree for a simple asset- liability management problem.

Then, for each event node, we have two branches; each branch is labelled by a conditional probability of occurrence, P(nk|ni), where ni = a(nk) is the immediate predecessor of node nk. Here, we have two nodes at time t = 1 and four at time t = 2, where we may rebalance our portfolio on the basis of the previous asset returns.

Finally, in the eight nodes corresponding to t = 3, the leaves of the tree, we just compare the terminal wealth with the liability and evaluate the utility function. Each node of the tree is associated with the set of asset returns during the corresponding time period. A scenario consists of an event sequence, i.e., a sequence of nodes in the tree, along with the associated asset returns. We have 8 scenarios in Fig. 4. For instance, scenario 2 consists of the node sequence (n0, nl, n3, n8). The probability of each scenario depends on the conditional probability of each node on its path. If each branch at each node is equally probable, i.e., the conditional probabilities are always 1/2, each scenario in the figure has probability ps = 1/8, for s = 1, ...,8. The branching factor may be arbitrary in principle; the more branches we use, the better our ability to model uncertainty; unfortunately, the number of nodes grows exponentially with the number of stages, as well as the computational effort.

At each node in the tree, we must make a set of decisions. In practice, we are interested in the decisions that must be implemented here and now, i.e., those corresponding to the first node of the tree; the other (recourse) decision variables are instrumental to the aim of devising a robust plan, but they are not implemented in practice, as the multistage model is solved on a rolling-horizon basis. This suggests that, in order to model the uncertainty as accurately as possible with a limited computational effort, a possible idea is to branch many paths from the initial node, and less from the subsequent nodes. Each decision at each stage may depend on the information gathered so far, but not on the future; this requirement is called a non-anticipatively condition. Essentially, this means that decisions made at time t must be the same for scenarios that cannot be distinguished at time t.

To build a model ensuring that the decision process makes sense, we can associate decision variables with nodes in the scenario trees and write the model in a way that relates each node to its predecessors.

Let us now introduce the following numerical data:

• The initial wealth is W^o=50.

• The target liability is L^s100.

• There are three assets, say, stocks A and B, and bonds; hence, I = 3.

• In the scenario tree of Fig. 4. we have up- and down-branches; in the (lucky) up-branches, total return is 1.28 for stock A, 1.40 for stock B and 1.20 for bonds; in the (bad) down-branches, total return is 1.08 for stock A, 0.99 for stock B and 1.12 for bonds. We see that bonds play the role of safer assets, and stocks B are very risky assets here. According to Barberies (2000), when asset returns are modelled as i.i.d. the mean and variance of cumulative log returns grow linearly with the investor's horizon.

• The reward rate q for excess wealth above the target liability is 1.

• The penalty rate r for the shortfall below the target liability is 4.

Let us introduce the following notation:

• N is the set of event nodes; in our case

N = {n0, n1, n2, …, n14}

• Each node n Є N, apart from the root node n0, has a unique direct predecessor node, denoted by a(n): for instance, a(n3) = n

• There is a set S ∩N of leaf (terminal) nodes; in our case

S = {n7, …, n14}

• For each node s Є S we have surplus and shortfall variables ws+ and ws_, related to the difference between terminal wealth and liability.

• There is a set T ∩ N of intermediate nodes, where portfolio rebalancing may occur after the initial allocation in node no; in our case

T = {n1, n2, …, n6}

• For each node n Є{n0} U T there is a decision variable x_{in}, expressing the money invested in asset i at node n.

With this notation, the model may be written as follows:

$\max \sum_{s\in S}(\pi^s)(qw_+^s - rw_-^s$-------------------------------(1)

such that: $\sum_{t=1}^{I} x_0^{no} = W^0$--------------------------------(2)

$\sum_{t=1}^{I} R_t^n x_t^{a(n)} = \sum_{t=1}^{I} x_t^n,\ \forall n \in \tau$------------------------(3)

$\sum_{t=1}^{I} R_t^s x_t^{a(s)} = L^s + w_+^s - w_-^s, \forall s \in S$ -----------------(4)

$x_t^n, w_+^s, w_-^s \geq 0$ ---(5)

where R_i^n is the total return for asset i during the period that leads to node n, and π^s is the probability of reaching the terminal node s Є S; this probability is the product of all the conditional probabilities on the path that leads from root node no to leaf node s.

NUMERICAL RESULTS

a. Piecewise Linear Utility Function: Recall that when asset returns are modelled as i.i.d., the mean and variance of cumulative log returns grow linearly with the investor's horizon (Barberies,2000). This leads an investor who rebalances his portfolio to choose the same asset allocation (see Table 1). These results suggest that analyses of dynamic strategies in which the uncertainty in not accurately represented should be interpreted with some caution. A possible solution is to branch many paths from the initial node, and less from the subsequent nodes or a more accurate representation of utility with more linear pieces.

Table 1. Investment strategy for a simple ALM problem with piecewise utility function.

Node	Stock A	Stock B	Bonds
n0	16.89	33.12	0
n1	67.97	0	0
n2	0	51.02	0
n3	0	0	87.01
n4	23.12	50.29	0
n5	0	71.43	0
n6	0	50.51	0

The Nonlinear Utility Functions

When we are approximating a nonlinear utility function by a piecewise linear function, the portfolio is not diversified and the wealth in the last time period is allocated to one asset (see Table 1). Actually, this alternative may imply "local" risk neutrality, so that we only care about expected return.

The uncertainty about the parameters may change over time. Therefore, the investment opportunity set perceived by the investor may change over time. To study the importance of uncertainty in a dynamic context we use the nonlinear programming model. As a result, the investor's are suggested to allocate their wealth in all assets (see Table 2). Using nonlinear utility function the objective of the optimization problem becomes:

$\max \sum_{s\in S}(\pi^s)\{(w_+^s)^2 - (w_-^s)^2$---------------------------(6)

Table 2. Investment strategy for a simple ALM problem with nonlinear utility function

Node	Stock A	Stock B	Bonds
n0	0	50	0
n1	0	70	0
n2	21.96	11.70	15.84
n3	0	98	0
n4	39.52	0	29.78
n5	35.37	4.48	23.65
n6	23.53	21.15	8.36

CONCLUSION

In the paper special emphasis was put on the shape of the investors' payoff functions in asset price equilibrium. The assumption of a linear utility function may imply "local" risk neutrality, so that we only care about expected return, resulting allocation of the wealth to one asset. When the asset returns are models as i.i.d. with piecewise utility function, then regardless of investment horizon an investor who rebalances his portfolio is suggested to choose the same asset allocation. On the other hand, when the nonlinear utility function is used the investor's are suggested to allocate their wealth in all assets. The results presented here suggest that portfolio calculations can be seriously misleading if the allocation framework ignores the fact that the uncertainty in not accurately represented.

In our paper we have assumed that the liabilities must be met, and this is a very hard constraint. If extreme scenarios are included in the formulation, it may well be the case that the model above is infeasible. Therefore, the formulation should be relaxed in a sensible way; we could consider the possibility of borrowing cash; we could also introduce suitable penalties for not meeting the liabilities. In principle, we could also require that the probability of not meeting the liabilities is small enough; this leads to chanceconstrained formulations, for which we refer the reader to the literature (Campbell JY, Viceira LM 2002, Heitsch H, Römisch W 2003, Hochreiter R, Pflug GC 2007, Klaassen P 2002, Liu J 2007, Wallace SW, Ziemba WT (eds) 2005).

REFERENCES:

Kusy, M.I., Ziemba, W.T. (1986): "A Bank Asset and Liability Management Model," Operations Research XXXIV: 356-376.

Samuelson P.A. (1969) Lifetime portfolio selection by dynamic stochastic programming. Rev Econ Stat 51:239–246.

Merton R.C. (1969): Lifetime portfolio selection under uncertainty: the continuous-time case. Rev Econ Stat 51:247–257.

Merton RC (1971) Optimum consumption and portfolio rules in a continuous-time model. J Econ Theory 3:373–413.

Markowitz H.M. (1952): "Portfolio selection". Journal of Finance, 7:77-91.

Blomvall J, Lindberg P.O. (2002): A Ricatti-based primal interior point solver for multistage stochastic programming. Eur J Oper Res 143:452–461.

Gondzio J, Grothey A. (2007): Solving nonlinear portfolio optimization problems with the primal-dual interior point method. Eur J Oper Res 181:1019–1029.

Wallace S.W., Ziemba W.T. (eds.). (2005): Applications of stochastic programming. MPS-SIAM book series on optimization.

Campbell J.Y., Chan Y.L., Viceira L.M. (2003): A multivariate model of strategic asset allocation. J Financ Econ 67:41–80.

Cariño, D., Ziemba, W. T. (1994): "The Russel-Yasuda Kasai Model: An Asset/Liability Model for a Japanese Insurance Company Using Multistage Stochastic Programming", Interfaces, Edelman Prize issue, 29-49.

Cariño, D., and Ziemba, W.T. (1998): "Formulation of the Russell-Yasuda Kasai Financial Planning Model", Operations Research, 46:4, 433-449.

Cariño, D., Myers, D., and Ziemba, W.T. (1998): Concepts, Technical Issues and Uses of the RussellYasuda Kasai Financial Planning Model, Operations Research, 46:4,450-462.

Gondzio J., Kouwenberg R. (2001): High performance computing for asset-liability management. Oper Res 49:879–891.

Zenios S.A. (1999): High-performance computing for financial planning: the last ten years and the next. Parallel Comput 25:2149–2175.

Ziemba, W.T., Mulvey, J.M., (eds.). (1998): Worldwide Asset and Liability modeling. Cambridge University Press.

Pflug, G.C., Swietanowski, A. (2000): Asset-liability optimization for pension fund management. In: Operations Research Proceedings, Springer, Heidelberg.

Geyer A., Ziemba W.T. (2007): The Innovest Austrian pension fund financial planning model InnoALM. Oper Res 56(4):797–810.

Geyer A., Hanke M., Weissensteiner A. (2007): Life-cycle asset allocation and optimal consumption using stochastic linear programming.Working paper, Vienna Graduate School of Finance and University of Innsbruck, available online: http://papers.ssrn.com/sol3/papers.cfm?abstract_id=952956.

Birge J.R. and Louveaux F. (1997): Introduction to Stochastic Programming, Springer-Verlag, New York.

Barberies N.C. (2000): Investing for the long run when returns are predictable. J.Finance 55:pp225-264.

Campbell JY, Viceira LM (2002) Strategic asset allocation. Oxford University Press, USA.

Heitsch H., Romisch W. (2003): Scenario reduction algorithms in stochastic programming. Comput Optim Appl 24:pp187-206.

Hochreiter R., Pflug G.C. (2007): Financial scenario generation for stochastic multi-stage decision as facility location problems. Ann Oper Res 152: pp257-272.

Klaassen P. (2002): Comment on generating scenario tress for multistage decision problems. Mange Sci 48:pp 1512-1516.

Liu J. (2007): Portfolio selection in stochastic environments. Rev Financ Stud 20:pp 1-39.